4) $xy^2z^2=1$ $\quad z-z_0=$

tangent

farthest from the origin.

$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$

$y_0^2z_0^2(x-x_0)+2x_0y_0z_0^2(y-y_0)+2x_0y_0^2z_0(z-z_0)=0$

$xy_0^2z_0^2-x_0y_0^2z_0^2+2x_0y_0z_0^2y-2x_0y_0^2z_0^2+2x_0y_0^2z_0z-2x_0y_0^2z_0^2=0$

$xy_0^2z_0^2+2x_0y_0z_0^2y+2x_0y_0^2z_0z=5$

$Ax+By+Cz+D=0 \quad (x_1,y_1,z_1)$

$$D=\frac{|Ax_1+By_1+Cz_1+D|}{\sqrt{A^2+B^2+C^2}}=\frac{5}{\sqrt{y^4z^4+4x^2y^2z^4+4x^2y^4z^2}}$$

$$D^2=\frac{25}{y^4z^4+4x^2y^2z^4+4x^2y^4z^2}$$

$$=\frac{25}{y^4\left(\frac{1}{xy^2}\right)^2+4x^2y^2\left(\frac{1}{xy^2}\right)^2+4x^2y^4\left(\frac{1}{xy^2}\right)}$$

$$f(x,y)=\frac{25}{\frac{1}{x^2}+\frac{4}{y^2}+4xy^2}$$

$$f_x=\frac{-25}{\left(\frac{1}{x^2}+\frac{4}{y^2}+4xy^2\right)^2}\left(\frac{-2}{x^3}+4y^2\right)x^3y^2=0$$

$$f_y=\frac{-25}{\left(\frac{1}{x^2}+\frac{4}{y^2}+4xy^2\right)^2}\left(\frac{-8}{y^3}+8xy\right)=0$$

$1=2x^3y^2$

$\frac{2}{x^3}=4y^2$

$1=xy^4 \quad x=\frac{1}{y^4}$

$\frac{1}{y^3}=xy$

$1=2y^{-12}y^2$

$y=\pm\sqrt[10]{2}$

$x=2^{-\frac{2}{5}}$

$z^2=\frac{1}{xy^2}=2^{\frac{2}{5}}2^{-\frac{1}{5}}$

$=2^{\frac{1}{5}}$

$z=\pm\sqrt[10]{2}$